GDP Per Capita in Africa before the Global Financial Crisis: Persistence, Mean Reversion and Long Memory Features

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This paper examined the long memory features of GDP per capita data before the global financial crisis, using a sample of 26 African countries. The study employed fractional integration and tested the stability of the differencing parameter across the sample period for each country. The results indicated that most of the countries' GDP series were I(1) or higher. Evidence of mean reversion was observed in 10 countries where the disturbances were autocorrelated. There was strong evidence against mean reversion in the remaining 16 countries. The results also indicated that the fractional differencing parameter was stable in 17 countries, while the presence of structural breaks was investigated in the remaining 9 countries.

Keywords: Africa, GDP, Global financial crisis, Long memory, Persistence.
JEL Classification: C22, E23

1.0 Introduction

Long memory and fractional integration are features commonly observed in macroeconomic time series. In the context of developed countries fractionally integrated or I(d) models have been widely employed to describe the behaviour of GDP, real GDP and GDP per capita series in Diebold and Rudebusch (1989), Haubrich and Lo (1991), Sowell (1992), Beran (1994), Silverberg and Verspagen (1999), Michelacci and Zaffaroni (2000), Mayoral (2006) and Caporale and Gil-Alana (2011) among many others. Many of the above papers deal with convergence and its different variations (real convergence, beta-convergence, etc.), testing this hypothesis by looking at the order of integration of the series. Thus, for I(d) models, if $d < 1$, mean reversion is obtained and convergence is satisfied. On the other hand, if $d = 1$ or $d > 1$, convergence is clearly rejected. The empirical results, at least for the developed countries, are mixed: some authors found evidence of unit roots in real output based on standard I(1) techniques (see Mankiw *et al.*, 1992, Quah, 1993, and Sala-i-Martin, 1996). Their approach is based on testing for convergence in a regression of the first differences of per capita output.

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Michelacci and Zaffaroni (2000) have pointed out that this finding cannot be reconciled with the other stylized facts of unit roots in output (see Nelson and Plosser, 1982), and a fairly smooth trend of output per capita in the OECD economies (see Jones, 1995). They show that US per capita output is well represented by a mean-reverting long memory I(d) process with $0.5 \leq d < 1$, where d is the fractional integration parameter.

This approach is criticised in Silverberg and Verspagen (2000) who concluded that the methods are biased in small samples. Instead, Silverberg and Verspagen (2000) use maximum likelihood approaches, and provide evidence that fractional integration in the range [0.5, 1), which is a key result in the paper by Michelacci and Zaffaroni (2000), disappears when these more appropriate methods are used. In another recent paper, Mayoral (2006) examined annual real GNP and GNP per capita in the US for the time period 1869-2001, using several parametric and semi-parametric methods. The results, though slightly different given the technique used, provide evidence that the orders of integration lie in the interval [0.5, 1), implying nonstationarity, high persistence and mean-reverting behaviour.

The economic literature on integration theory assumes two distinct positions relative to the process of country's growth and the catching up hypothesis. The first is the theory of country divergence which argues that a higher integration towards a single currency is expected to increase factor mobility which can be in favour of the prosperous countries. Therefore, concentration of economic activity to these attractive countries which dispose more developed markets and higher level of industrialization can create additional difficulties to the less developed country and delay their catching up process. The theory of regional convergence argues that a higher integration will attenuate the initial regional disparities and in the long run leads to regional convergence rather than divergence. It further argues that, for example, the United States of America shows lower regional disparities than the European Union as a result of deeper economic integration which includes the monetary integration and common currency. Marques and Soukiazis (1998) investigated per capita income convergence across countries and regions in the European Union (EU) using the "sigma" and "beta" convergence approach. They found different convergence level for each of the data subsamples.

For developing countries, the literature on convergence in GDP is scarce. Real convergence in some emerging countries has been examined by Cunado *et al*.

(2004, 2007). Jones (2002) considers the convergence in GDP for a group of low-income countries in Africa using both cross-sectional and time series approaches. He concludes that there is convergence among the Economic Community of West African States (ECOWAS) between 1960 and 1990. He further states that the speed at which wealthy countries can catch up with the poor in terms of growth is very slow. Charles *et al.* (2008) consider real GDP per capita in the common market for Eastern and Southern Africa from 1950 to 2003 using panel unit root tests. Using this methodology, they find no evidence of stochastic or conditional convergence in the regions considered. Joseph (2010) investigates the convergence between a slow growing economy and a fast growing economy using the case of Ghana as an African country and UK as a Western European country, accepting the hypothesis of convergence.

This paper, to the best of our knowledge, is the first to apply fractional integration techniques in studying convergence among African countries. We consider the convergence hypothesis in a group of 26 African countries using their GDP per capita data. Following this section, section 2 briefly describes the main ideas of the I(d) models. Section 3 presents the data. Section 4 displays the empirical results and Section 5 deals with the robustness of the results presented. Section 6 concludes the paper.

2.0 The I(d) model

To correctly determine the order of integration in time series is crucial from both economic and statistical viewpoints. First, we need to introduce some definitions. Given a covariance stationary process $\{u_t, t = 0, \pm 1, \ldots\}$, with autocovariance function $E(u_t - Eu_t)(u_{t-j} - Eu_t) = \gamma_j$, we say that u_t is integrated of order 0 if

$$\lim_{T \to \infty} \sum_{j=-T}^{T} \left| \gamma_j \right| < \infty.$$

Alternatively, assuming that u_t has an absolutely continuous spectral distribution function, so that it has a spectral density function, denoted by $f(\lambda)$, and defined in terms of the autocovariances as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \le \pi,$$

we say that u_t is I(0) if the spectral density function $f(\lambda)$ is positive and finite at all frequencies (λ) in the spectrum, i.e.,

$$0 < f(\lambda) < \infty, \quad \textit{for all } \lambda.$$

The I(0) models are sometimes called "short memory" due to the fact that, if there is a degree of association between the observations, this is short and tends to disappear fast as the distance between the observations increases. Standard I(0) processes are the white noise and the stationary ARMA specifications.

Having said this, a process x_t is said to be I(d) (and denoted by $x_t \approx I(d)$) if:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \tag{1}$$

where L is the lag operator ($Lx_t = x_{t-1}$), d is a real value, and u_t is an I(0) process as defined just above.

The I(d) model with d > 0 belongs to a broader class of models called "long memory", which are characterized by the infinite sum of the autocovariances is infinite, or, alternative, in the frequency domain, because the spectral density function is unbounded at some point(s) in the spectrum.

The fractional differencing parameter d plays a crucial role from both economic and statistical viewpoints. Thus, if d = 0 in (1), $x_t = u_t$, the process is I(0) and it could be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially; however, it could decay at a much slower rate than exponentially (in fact, hyperbolically) if d is positive. Moreover, if $0 < d < 0.5$, x_t is covariance stationary, but its lag-j autocovariance γ_j decreases very slowly, at the rate of j^{2d-1} as $j \to \infty$, and so the γ_j are absolutely non-summable. The variable x_t is then said to have long memory given that $f(\lambda)$ is unbounded at the origin, i.e.,

$$f(\lambda) \to \infty, \quad as \quad \lambda \to 0^+.$$ [1]

[1] The origin of these processes is in the 1960s, when Granger (1966) and Adelman (1965) pointed out that most aggregate economic time series have a typical shape where the spectral density increases dramatically as the frequency approaches zero. However, differencing the data frequently leads to over-differencing at the zero frequency.

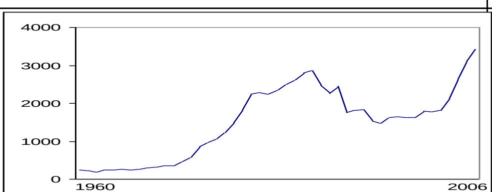
Also, as d in (1) increases beyond 0.5 and through 1 (the unit root case), x_t can be viewed as becoming "*more nonstationary*" in the sense, for example, that the variance of the partial sums increases in magnitude. Processes of the form given by (1) with positive non-integer d are called fractionally integrated, and when u_t is ARMA(p, q), x_t is known as a fractionally ARIMA (or ARFIMA) model. This type of model provides a higher degree of flexibility in modelling low frequency dynamics which is not achieved by non-fractional ARIMA models. Another interesting distinction is the case of d < 1 as opposed to $d \geq 1$. In the former case, the process is mean reverting with shocks disappearing in the long run. On the contrary, if $d \geq 1$, mean reversion does not occur, and the effect of the shocks persists forever in the series. This is relevant in the context of GDP series noting that a shock in the series may have a different effect (in the short and in the long run) depending on the value of the fractional differencing parameter d.

3.0 The data

The data used in this work are based on annual GDP per capita from 1960 to 2006, obtained from International Monetary Fund (IMF). We intentionally avoided periods of global financial crisis, since these would affect the results negatively.

Figure 1: Plots of original time series data

ALGERIA



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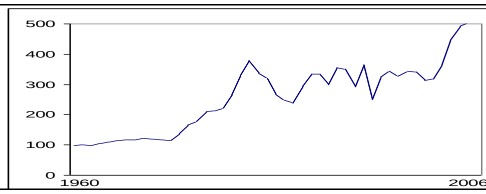

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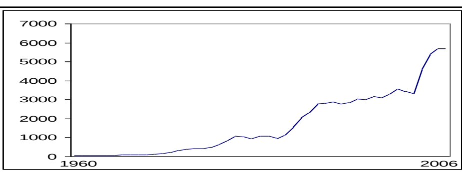
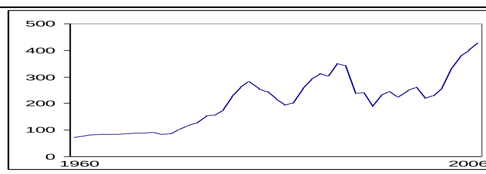
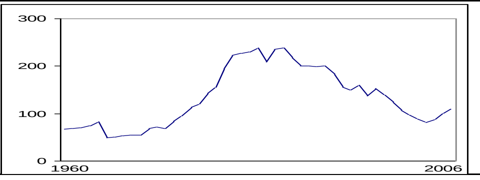


BURKINA FASO

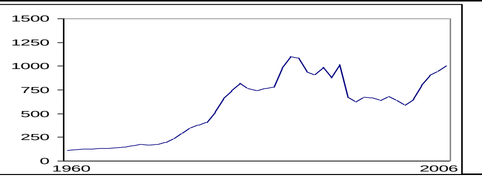


BURUNDI

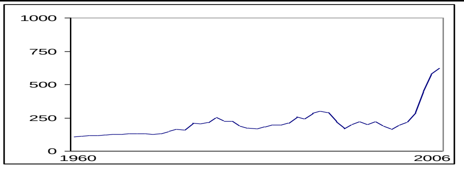
CAMEROON



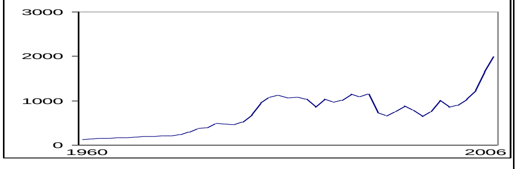
CHAD



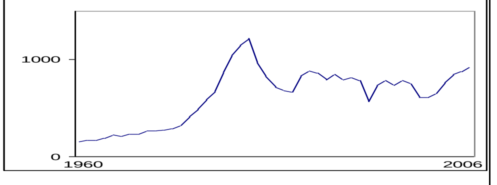
CONGO



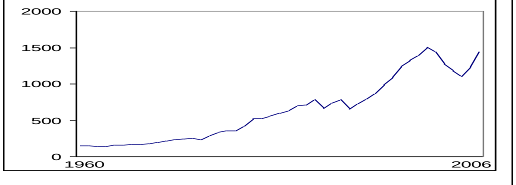
COTE D' IVOIRE



EGYPT



GABON



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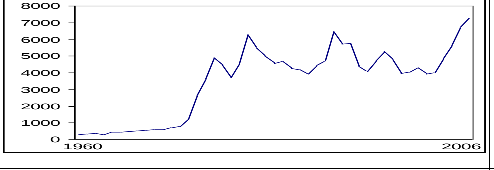

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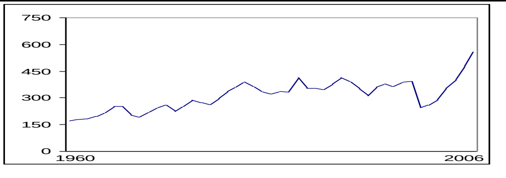

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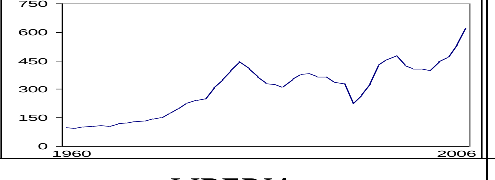
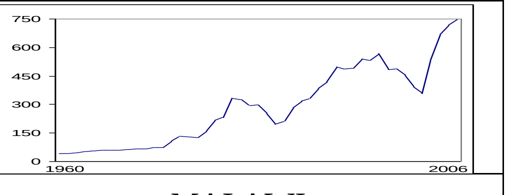
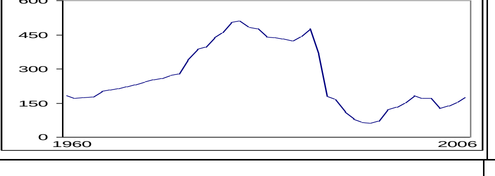


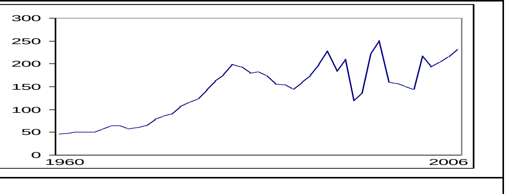
LIBERIA



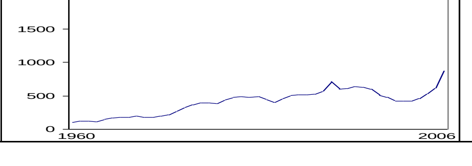
MALAWI

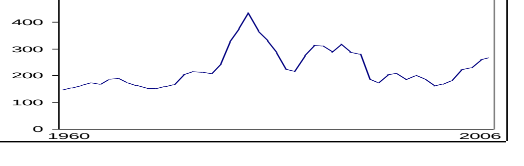


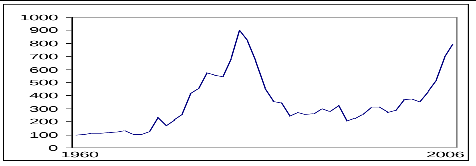
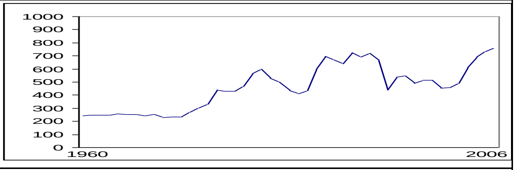
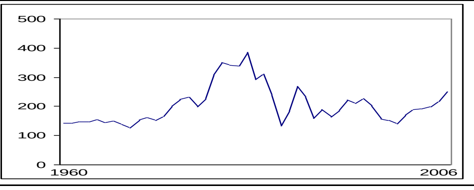
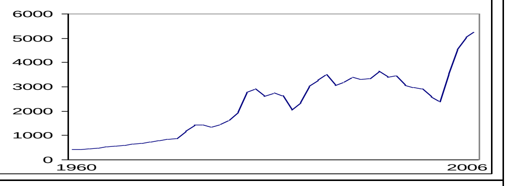
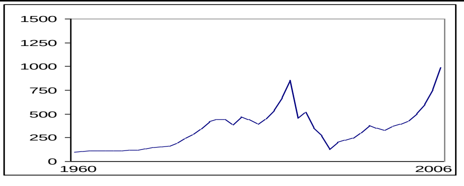
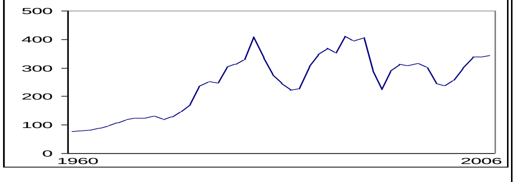
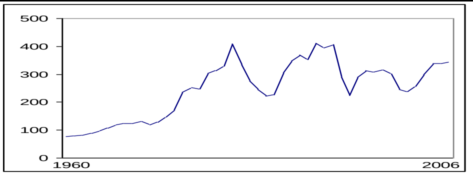
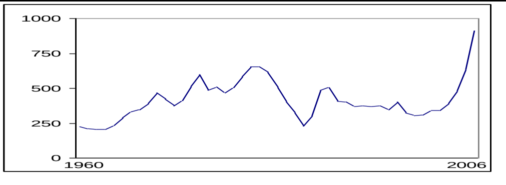
MAURITANIA



NIGER





NIGERIA	SENEGAL
	
SIERRA LEONE	SOUTH AFRICA
	
SUDAN	TOGO
	
UGANDA	ZAMBIA
	

Plots of the time series data are displayed in Figure 1. Along this figure we can distinguish two different patterns across the countries. On the one hand we have countries displaying a relatively constant increase across the sample. Within this group we could include countries such as Benin, Botswana, Burkina Faso, Egypt, Gabon, Ghana, Kenya, Lesotho, Malawi, Mauritania, South Africa, Senegal, Togo and Uganda. On the other hand, the remaining countries (Algeria, Burundi, Cameroon, Chad, Congo, Cote d' Ivoire, Liberia, Niger, Nigeria, Sierra Leone, Sudan and Zambia) are characterized by an oscillating pattern with a substantial decrease during the 90s, and a posterior increase during the last years in the sample.[2] The first 20 sample autocorrelation values of the first differences for each series were generated,

[2] Note, the results of sample autocorrelations were obtained graphically and are available on request. Also, the Fortran codes written to plot the graphs and compute the fractional integration parameters are available on request from the authors.

and we observed a rapid decrease in the majority of the cases, with most of the values found within the 95% confidence interval. However, in many of the series we also observe some significant values away from zero which may suggest that fractional differentiation (with d smaller or higher than 1) may be more appropriate than first differences in some cases.

4.0 The empirical results

The first thing we do in this section is to estimate the fractional differencing parameter d in the model given by equation (1), where x_t can be the errors in a regression model of form:

$$y_t = \alpha + \beta t + x_t, \qquad t = 1, 2, ... \tag{2}$$

where y_t is the observed GDP time series; α and β are the coefficients corresponding to the intercept and a linear time trend, and based on equation (1), u_t is supposed to be I(0). For this purpose we employ a Whittle function in the frequency domain. Along with the estimates of α, β and the fractional differencing parameter d, we also compute the confidence bands of the non-rejection values of d using the Lagrange Multiplier (LM) procedure of Robinson (1994). This method tests the null hypothesis:

$$H_o : d = d_o, \tag{3}$$

in (1) and (2) for a grid of real-values d_o. Thus, the null model tested is:

$$y_t = \alpha + \beta t + x_t; \qquad (1 - L)^{d_o} x_t = u_t, \qquad t = 1, 2, ...,$$

with I(0) u_t.

Table 1 displays the estimates of d (along with the 95% confidence band of the non-rejection values of d using Robinson's (1994) method) in the model given by equations (1) and (2) under the assumption that u_t in (1) is a white noise process. Thus, all the time dependence in the process is determined by the fractional differencing parameter d. We report the estimates of d for the three standard cases examined in the literature, i.e. a) the case of no regressors (α = β = 0 in (2)); b) an intercept (α unknown and β = 0); and c) an intercept with a linear time trend.

Table 1: Estimates of d based on white noise disturbances

	No regressors	An intercept	A linear time trend
ALGERIA	1.31 (1.14, 1.53)*	1.3 (1.12, 1.52)*	1.29 (1.12, 1.52)*
BENIN	0.88 (0.64, 1.16)	0.79 (0.59, 1.09)	0.81 (0.57, 1.10)
BOTSWANA	1.19 (1.00, 1.61)*	1.19 (1.00, 1.62)*	1.22 (1.00, 1.64)*
BURKINA FASO	1.16 (0.91, 1.46)	1.13 (0.83, 1.47)	1.13 (0.86, 1.48)
BURUNDI	1.19 (1.04, 1.41)*	1.22 (1.09, 1.42)*	1.22 (1.09, 1.42)*
CAMEROON	1.05 (0.86, 1.32)	1.04 (0.85, 1.32)	1.04 (0.86, 1.31)
CHAD	1.41 (1.24, 1.63)*	1.46 (1.25, 1.75)*	1.46 (1.25, 1.76)*
CONGO	1.27 (0.99, 1.58)	1.22 (0.87, 1.55)	1.22 (0.94, 1.54)
COTE D' IVOIRE	1.25 (0.99, 1.58)	1.25 (0.99, 1.61)	1.25 (0.99, 1.61)
EGYPT	1.29 (0.91, 1.82)	1.17 (0.88, 1.75)	1.18 (0.87, 1.74)
GABON	1.05 (0.76, 1.44)	1.04 (0.73, 1.43)	1.04 (0.79, 1.43)
GHANA	1.18 (0.92, 1.49)	0.96 (0.36, 1.44)	1 (0.79, 1.43)
KENYA	1.35 (1.03, 1.71)*	1.31 (0.85, 1.72)	1.3 (0.93, 1.72)
LESOTHO	1.16 (0.85, 1.55)	1.15 (0.82, 1.57)	1.16 (0.82, 1.57)
LIBERIA	1.25 (1.08, 1.52)*	1.36 (1.17, 1.68)*	1.36 (1.17, 1.68)*
MALAWI	**0.51 (0.35, 0.95)*** **	**0.59 (0.47, 0.90)*** **	**0.55 (0.36, 0.90)*** **
MAURITANIA	1.44 (1.04, 1.81)	1.27 (0.51, 1.71)	1.24 (0.80, 1.68)
NIGER	1.17 (0.93, 1.50)	1.23 (0.92, 1.65)	1.22 (0.93, 1.65)
NIGERIA	1.31 (1.09, 1.61)*	1.29 (1.06, 1.61)*	1.29 (1.06, 1.61)*
SENEGAL	1.02 (0.77, 1.34)	0.99 (0.71, 1.37)	1 (0.73, 1.37)
SIERRA LEONE	0.95 (0.74, 1.27)	0.86 (0.64, 1.20)	0.87 (0.66, 1.20)
SOUTH AFRICA	1.2 (0.86, 1.61)	1.19 (0.76, 1.65)	1.19 (0.83, 1.65)
SUDAN	1.07 (0.83, 1.34)	1.01 (0.70, 1.30)	1.02 (0.80, 1.30)
TOGO	1.01 (0.72, 1.42)	1.02 (0.72, 1.45)	1.02 (0.75, 1.45)
UGANDA	1.12 (0.71, 1.54)	1.11 (0.63, 1.57)	1.11 (0.70, 1.57)
ZAMBIA	1.51 (1.22, 1.89)*	1.45 (1.01, 1.92)*	1.43 (1.04, 1.93)*

*: Evidence of I(d) with d > 1.

***: Evidence of mean reversion (I(d) with d < 1)

We see in Table 1 that independently of the regressors included in the model the orders of integration are relatively high in all cases. In fact, the estimated value of d is found to be higher than 1 in the majority of the cases. Evidence of mean reversion (i.e., an order of integration statistically significantly smaller than 1) is only found in a single case corresponding to Malawi.[3] In a number of countries (Benin, Sierra Leone, Ghana and Senegal) we also observe some estimates, which are below 1 though the I(1) null hypothesis (i.e. d = 1) cannot be rejected at conventional statistical levels. There is an

[3] Malawi is the only country where a significant negative value for the first sample autocorrelation was obtained and that was as a result of overdifferentiation of the first differenced data. Here, in Table 1, the result of *d* for the case of Malawi is consistent with the previous autocorrelations result.

additional group of countries where the unit root null cannot be rejected, including Burkina Faso, Cameroon, Congo, Cote d' Ivoire, Egypt, Gabon, Lesotho, Niger, South Africa, Sudan, Togo and Uganda. For the remaining countries (Algeria, Botswana, Burundi, Chad, Kenya, Liberia, Mauritania, Nigeria and Zambia) the unit root null is decisively rejected in favour of orders of integration which are above 1. These results are summarized in Table 2.

Table 2: Summary results from Table 1

Mean Reversion	I(1) behaviour	I(d) with d > 1
MALAWI	BENIN	ALGERIA
	BURKINA FASO	BOTSWANA
	CAMEROON	BURUNDI
	CONGO	CHAD
	COTE D' IVOIRE	KENYA*
	EGYPT	LIBERIA
	GABON	MAURITANIA*
	GHANA	NIGERIA
	LESOTHO	ZAMBIA
	NIGER	
	SENEGAL	
	SIERRA LEONE	
	SOUTH AFRICA	
	SUDAN	
	TOGO	
	UGANDA	

*: In these countries the unit root cannot be rejected in some cases.

The results presented so far do not take into account weak autocorrelation for the error term. This may produce potential biases in the estimated values of d. A common practice is to employ thee ARMA models for the error term u_t in (1) (Robinson, 1994). However, given the difficulty in determining the appropriate orders for the AR and MA polynomials, especially in small samples as is the case in the present work, we use a less conventional approach due to Bloomfield (1973). This is a non-parametric method, where the model is implicitly determined by the spectral density function, which is given by:

$$f(\lambda; \sigma^2) = \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^{m} \tau_r \cos(\lambda r) \right),$$

where $\sigma^2 = \mathrm{Var}(\epsilon_t)$ and m is an integer value describing the short run dynamics of the series. This model produces autocorrelations decaying exponentially as in the ARMA case, and it accommodates extremely well in the context of the tests of Robinson (1994) employed in this work.[4] Using this approach the results are presented in Table 3.

Table 3: Estimates of d based on Bloomfield (1973) disturbances

	No regressors	Intercept	Linear time trend
ALGERIA	1.43(0.29, 2.03)	1.30(0.32, 1.95)	1.23(0.48, 1.95)
BENIN	0.48 (0.17, 1.49)	0.52 (0.23, 1.41)	0.43 (-0.27, 1.39)
BOTSWANA	0.46 (0.29, 0.83) ***	0.49 (0.29, 0.90) ***	0.03 (-0.17, 0.53)
BURKINA FASO	0.36 (0.12, 1.45)	0.45 (0.15, 1.35)	0.36 (-0.16, 1.33)
BURUNDI	1.26 (0.89, 1.85)	1.35 (1.02, 1.85)*	1.34 (1.02, 1.82)*
CAMEROON	0.86 (0.48, 1.42)	0.86 (0.51, 1.40)	0.90 (0.54, 1.37)
CHAD	1.40 (0.81, 1.99)	1.21 (1.00, 1.64)*	0.41 (-0.17, 1.63)
CONGO	0.26 (0.03, 1.64)	0.33 (0.07, 1.54)	0.33 (-0.16, 1.51)
COTE D' IVOIRE	0.65 (0.23, 1.39)	0.64 (0.31, 1.38)	0.73 (0.34, 1.39)
EGYPT	0.63 (0.47, 0.96) ***	0.73 (0.58, 1.02)	0.30 (-0.14, 1.00)
GABON	0.38 (0.16, 1.05)	0.45 (0.22, 0.97)	0.47 (0.16, 1.02)
GHANA	0.07 (-0.03, 1.53)	0.16 (-0.07, 0.41) ***	-0.07 (-0.34, 0.96) ***
KENYA	0.32 (0.12, 1.78)	0.44 (0.16, 1.70)	0.35 (-0.10, 1.63)
LESOTHO	0.48 (0.27, 1.02)	0.51 (0.30, 0.94) ***	-0.27 (-0.77, 0.95) ***
LIBERIA	1.08 (0.75, 1.54)	1.03 (0.66, 1.53)	1.03 (0.67, 1.51)
MALAWI	0.24 (0.12, 0.45) ***	0.40 (0.19, 0.60) ***	0.19 (-0.10, 0.57) ***
MAURITANIA	0.34 (0.19, 2.44)	0.47 (0.28, 3.00)	0.46 (0.07, 3.05)
NIGER	0.74 (0.13, 1.38)	0.47 (0.14, 1.17)	0.53 (0.18, 1.17)
NIGERIA	0.89 (-0.05, 1.53)	0.58 (-0.08, 1.45)	0.83 (0.19, 1.44)
SENEGAL	0.25 (0.10, 1.23)	0.42 (0.18, 0.92) ***	0.25 (-0.16, 0.93) ***
SIERRA LEONE	0.46 (0.03, 1.07)	0.39 (0.03, 0.89) ***	0.43 (0.05, 0.89) ***
SOUTH AFRICA	0.30 (0.14, 0.61) ***	0.40 (0.18, 0.64) ***	-0.16 (-0.56, 0.53) ***
SUDAN	1.16 (-0.05, 2.14)	1.05 (-0.07, 2.20)	1.04 (-0.01, 2.15)
TOGO	0.28 (0.13, 0.90) ***	0.44 (0.23, 0.83) ***	0.40 (0.08, 0.86) ***
UGANDA	0.09 (-0.04, 0.99)***	0.19 (-0.06, 0.54) ***	-0.20 (-0.62, 0.71) ***
ZAMBIA	-0.02 (-0.20, 1.52)	-0.03 (-0.30, 1.45)	0.30 (-0.20, 1.33)

*: Evidence of I(d) with d > 1. ***: Evidence of mean reversion (I(d) with d < 1)

[4] See Gil-Alana (2004) for a paper dealing the model of Bloomfield (1973) in the context of Robinson's (1994) tests.

Table 4: Summary results from Table 2

Mean Reversion	I(1) behaviour	I(d) with d > 1
BOTSWANA	ALGERIA	BURUNDI
EGYPT	BENIN	CHAD
GHANA	BURKINA FASO	
LESOTHO	CAMEROON	
MALAWI	CONGO	
SENEGAL	COTE D' IVOIRE	
SIERRA LEONE	GABON	
SOUTH AFRICA	KENYA	
TOGO	LIBERIA	
UGANDA	MAURITANIA	
	NIGER	
	NIGERIA	
	SUDAN	
	ZAMBIA	

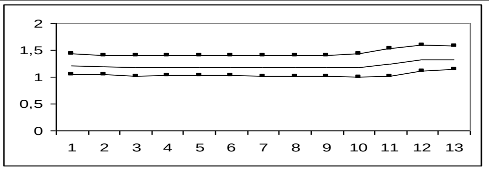
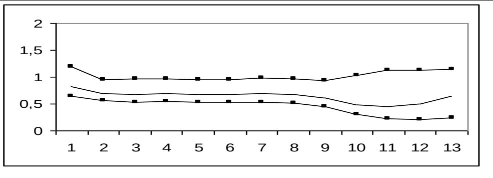
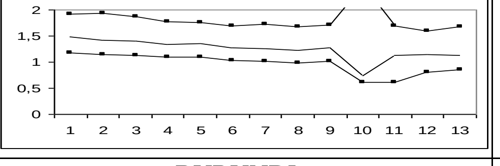
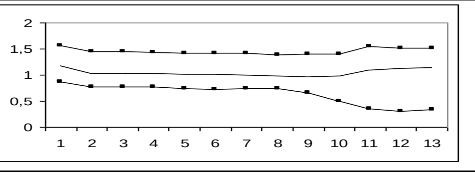
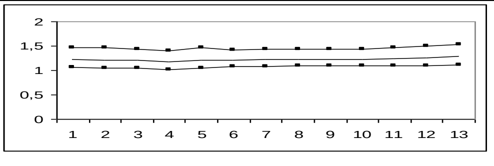
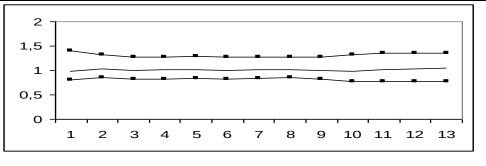
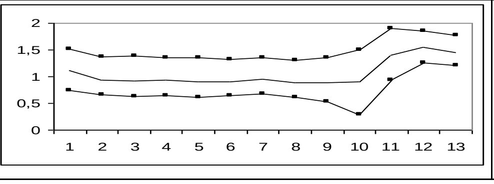
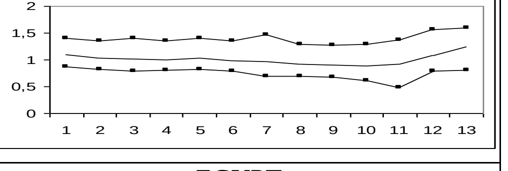
The first noticeable feature in this table is that the confidence intervals are very large implying that in some cases we cannot reject neither the I(0) nor the I(1) hypotheses. We also observe in Table 3 that the results differ in some cases depending on the specification of the deterministic terms. Thus, evidence of mean reversion is obtained for Botswana (in the cases of no regressors and with an intercept); Egypt (with no regressors); Ghana, Lesotho, Senegal and Sierra Leone (with an intercept, and with an intercept and a linear time trend); and also for Malawi, South Africa, Togo and Uganda (in all cases). For the remaining countries the results support the existence of unit roots or orders of integration above 1 (as is the case with Burundi and Chad). Table 4 summarizes the results displayed in Table 3. We see that mean reversion is obtained in the cases of Botswana, Egypt, Ghana, Lesotho, Malawi, Senegal, Sierra Leone, South Africa, Togo and Uganda. Thus, if autocorrelation is allowed, apart from Malawi, we observe seven additional countries showing estimates of d which are below 1and thus showing mean reverting behaviour.

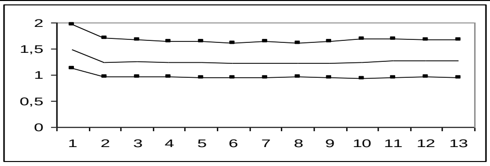
The results presented so far assume that the fractional differencing parameter has remained constant across the sample period. This is a strong assumption especially in the context of African countries. In the next section we examine this issue looking at the stability of d across the sample period.

5.0 Stability tests and robustness checking

Two approaches are implemented here. In the first, we estimate d, for each country, in a sample for the time period [1960 – 1994]; then we re-estimate d moving the sample one period forward, i.e., [1961 – 1995] and so on till the final subsample [1972 – 2006]. Thus, in all cases we consider subsamples of 35 observations each. In the second approach, we start with the same subsample as in the previous case, adding then one observation each time till the sample is completed, [1960 – 2006]. The results in terms of the estimation of d (with their corresponding 95% confidence intervals) are respectively displayed in Figures 2 and 3.

Figure 2: Results based on moving windows with 35 observations for each series moving recursively.

ALGERIA	BENIN
	
BOTSWANA	BURKINA FASO
	
BURUNDI	CAMEROON
	
CHAD	CONGO
	
COTE D' IVOIRE	EGYPT



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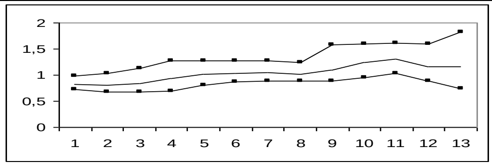
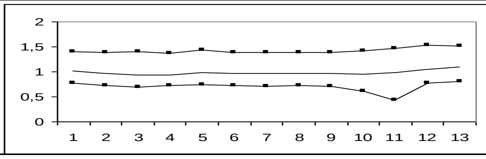
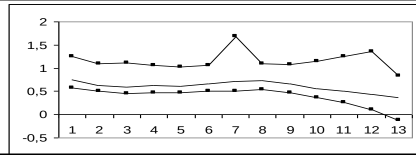


GHANA



KENYA



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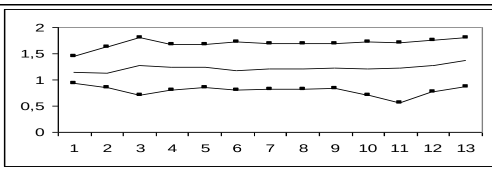
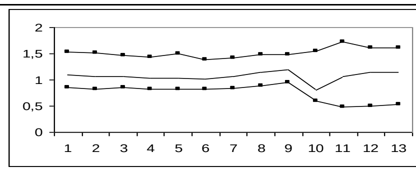


LIBERIA



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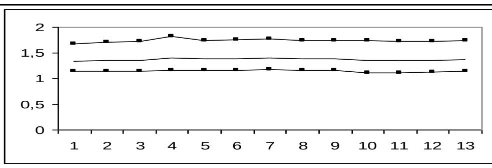

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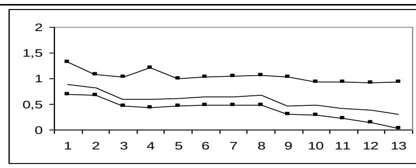

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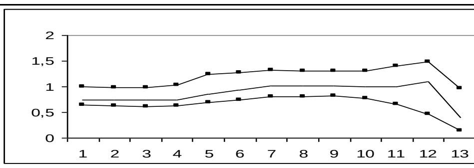

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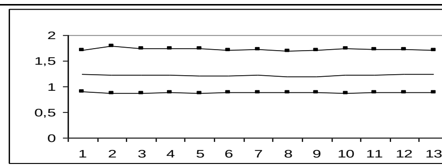
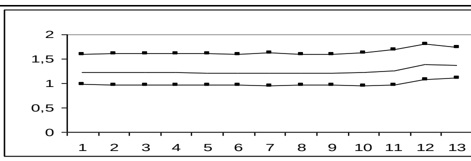
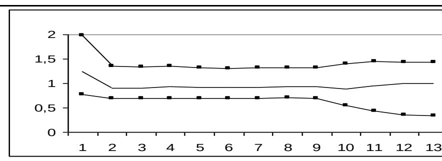


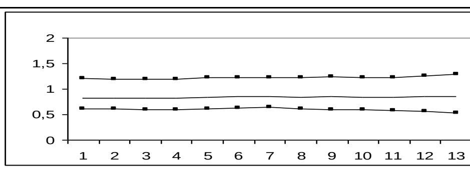
SENEGAL



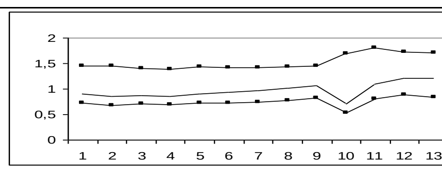
SIERRA LEONE



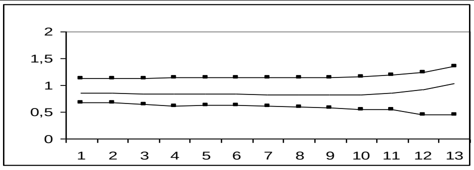
SOUTH AFRICA

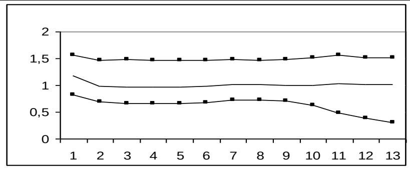


SUDAN



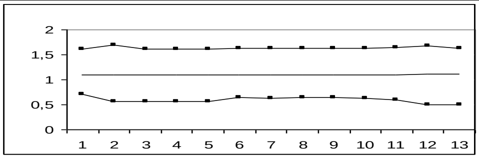
TOGO

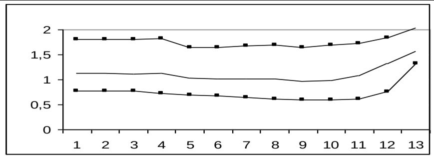




UGANDA

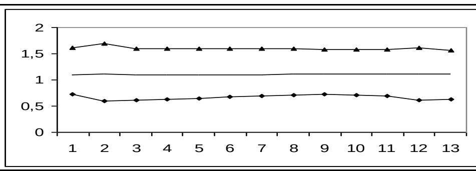
ZAMBIA

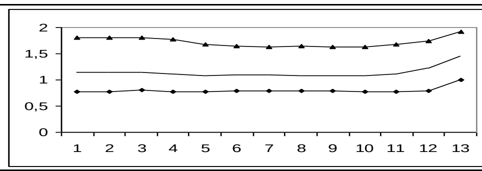




UGANDA

ZAMBIA



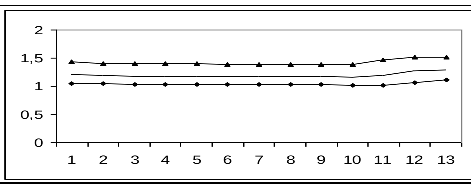


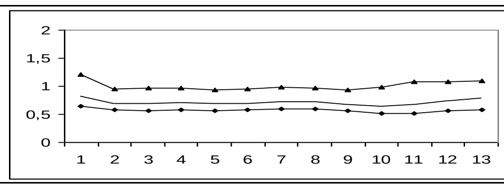
Looking at the results in Figure 2, we notice ten countries where the estimates of d remain stable across the sample. They correspond to Algeria, Burkina Faso, Burundi, Cameroon, Gabon, Kenya, Liberia, Niger, Nigeria and Sierra Leone. There are three additional countries (Cote d' Ivoire, Senegal and Togo) where we observe an initial decrease in the estimate of d though with a relative stable behaviour across the sample period. In six countries (Chad, Congo, Egypt, South Africa, Sudan and Zambia) we observe an increase in the value of d across the sample, and a general decrease is observed in countries such as Benin, Botswana, Ghana, Lesotho, Malawi and Mauritania.

Figure 3: Results based on moving windows adding one observation each time to the initial 35 samples

ALGERIA

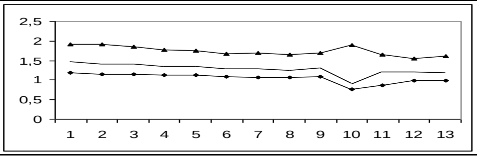
BENIN



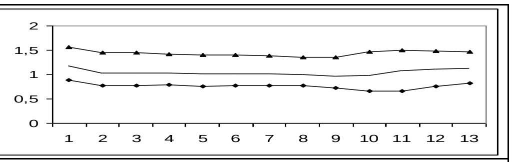


BOTSWANA

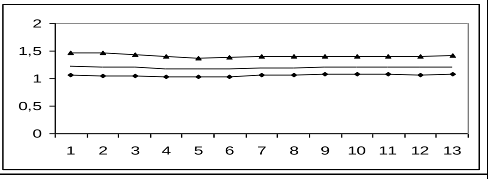
BURKINA FASO



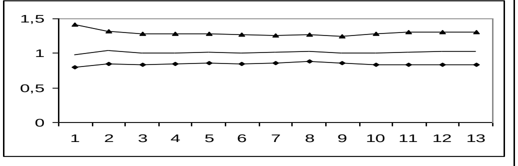
BURUNDI



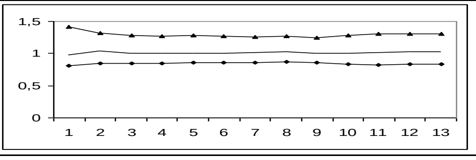
CAMEROON



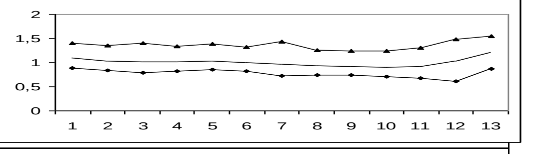
CHAD



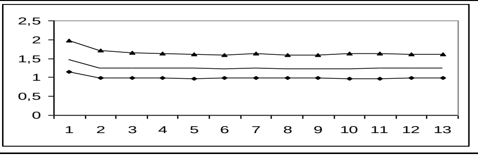
CONGO



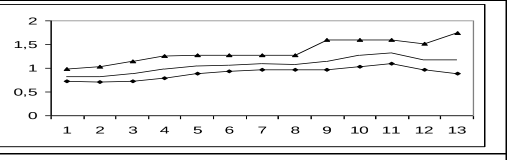
COTE D’ IVOIRE



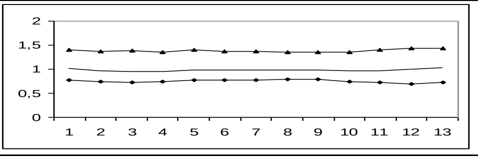
EGYPT



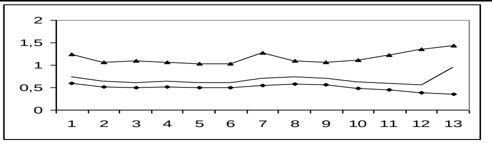
GABON



GHANA



KENYA



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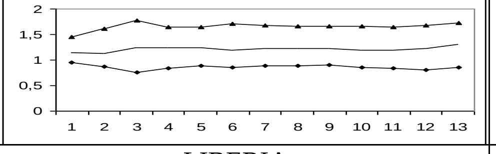

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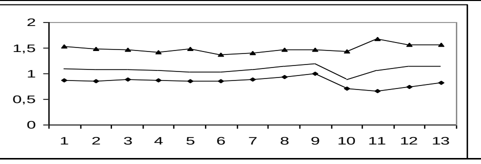

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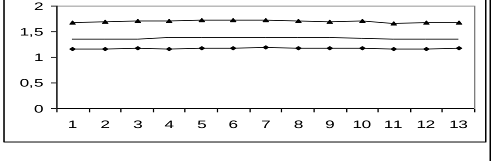

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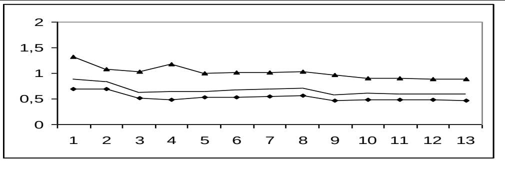

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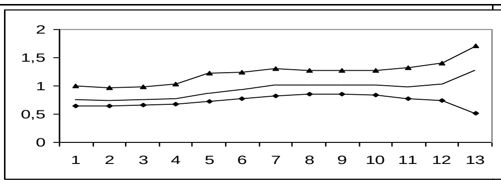
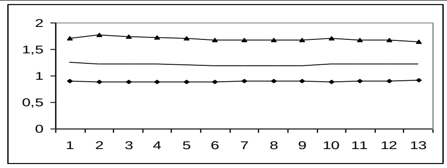


NIGERIA



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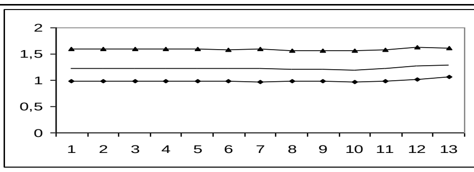
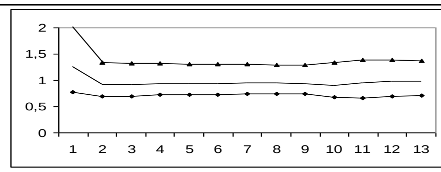
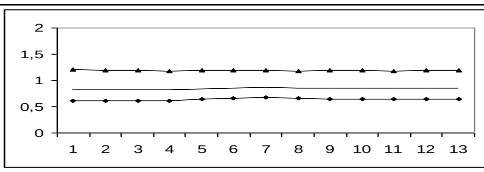


SIERRA LEONE



SOUTH AFRICA



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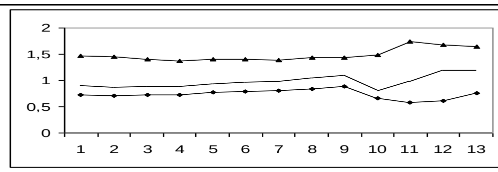
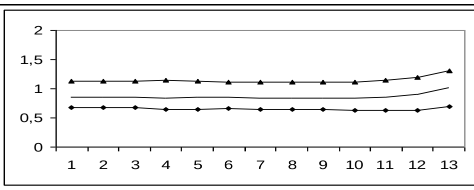


TOGO



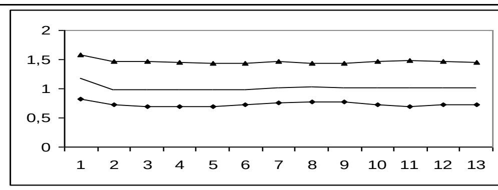


Figure 3 displays the estimates of d adding one observation each time. We notice in this table that countries like Algeria, Benin, Burkina Faso, Burundi, Cameroon, Chad, Cote d' Ivoire, Congo, Gabon, Kenya, Liberia, Niger, Nigeria, Senegal, Sierra Leone, Sudan and Togo, the estimates of d are very stable. These countries are the same as those reported in Figure 2 with the inclusion of Chad, Benin, Congo and Sudan. For the remaining countries, we observe a reduction in the estimation of d (Botswana, Lesotho and Malawi) and an increase for Egypt, Ghana, Mauritania, South Africa and Zambia, which is once more consistent with the results reported in Figure 2.

Due to the small sample sizes of the series examined, it is difficult to draw conclusions about the existence of breaks. Nevertheless, we have taken the eight series where we noticed unstable behaviour in the parameter d across Figure 3 and have looked deeper at its behaviour across the sample period.

Table 5: Potential breaks in the series

Countries	Break date(s)	d_1	d_2	d_3
BOTSWANA	2003	1.31 (1.09, 1.69)	0.9 (0.76, 1.89)	N/A
LESOTHO	2003	1.2 (1.00, 1.47)	0.88 (0.71, 1.44)	N/A
MALAWI	1996, 2002	0.84 (0.69, 1.08)	0.63 (0.52, 1.03)	0.58 (0.47, 0.96)
EGYPT	2002	1.08 (0.96, 1.27)	1.15 (0.96, 1.55)	N/A
GHANA	2006	0.56 (0.38, 1.35)	0.96 (0.36, 1.44)	N/A
MAURITANIA	2006	1.04 (0.74, 1.41)	1.27 (0.51, 1.71)	N/A
S. AFRICA	2003	1.09 (0.89, 1.43)	0.8 (0.66, 1.49)	N/A
ZAMBIA	2006	1.23 (0.79, 1.75)	1.45 (1.00, 1.92)	N/A

Table 5 displays in the second column the years of the breaks across the selected countries, which are Botswana, Lesotho, Malawi, Egypt, Ghana, Mauritania, South Africa and Zambia. It is observed that in all except one country (Malawi) there is one single break in the data, which takes place around 2002/03 in the majority of the series. Only for Ghana and Mauritania the breaks takes place in the final year of the sample period, 2006. For Malawi, two breaks are observed, one in 1996 and the other in 2002. If we focus now on the fractional differencing parameters we notice that for countries such as Botswana, Lesotho and South Africa, there is a reduction in the order of integration from values above 1 to values below 1 though the unit root cannot be rejected in the second estimates; for Malawi, there is a considerable reduction in the estimation of d, from 0.84 when using the first subsample (ending at 1996) to 0.63 (with the sample ending at 2002) and to 0.58 (and showing significant evidence of mean reversion) when using the whole sample period. For the remaining countries (Egypt, Ghana, Mauritania and Zambia) we observe an increase in the degree of persistence of the series with the sample.

6.0 Concluding Remarks

In this paper we examine the degree of persistence in the GDP per capita series in a group of 26 African countries using long range dependence techniques. We focus on annual data from 1960 to 2006 and the results indicate that most of the series are highly persistent with orders of integration equal to or higher than 1. The exceptions are Malawi (if the disturbances in the d-differenced process are uncorrelated) and Botswana, Egypt, Ghana, Lesotho, Senegal, Sierra Leone, South Africa, Togo and Uganda, along with Malawi if the disturbances are autocorrelated. In all these cases mean reversion may occur to some extent. The stability of the fractional differencing parameter across the sample is also investigated. The results here indicate that the parameter d is quite stable in the cases of Algeria, Benin, Burkina Faso, Burundi, Cameroon, Chad, Cote d' Ivoire, Congo, Gabon, Kenya, Liberia, Niger, Nigeria, Senegal, Sierra Leone, Sudan and Togo. In the remaining countries (Botswana, Egypt, Ghana, Lesotho, Malawi, Mauritania, South Africa and Zambia) the possibility of a structural break was considered, and evidence of breaks is found about 2002/03 in the majority of the cases, noticing a reduction in the degree of persistence for Botswana, Lesotho, South Africa and Malawi, and an increase for Egypt, Ghana, Mauritania and Zambia.

This work is limited by the small sample size of the annual time series data applied. In the future, one can consider applying the quarterly time series to examine the change in the persistence in GDP per capita before and after the global crisis.

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